P: ISSN NO.: 2394-0344
E: ISSN NO.: 2455-0817
Remarking
Vol-II * Issue-VIII* January- 2016

Fixed Point Theorem for Compatible Mapping in Cone Metric Space

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Abstract

In this paper are prove the fixed point theorem for compatible mapping in cone metric space including above result M. Chandra and B.E. Rhoades. The main purpose of this paper fixed point theorem for compatible mapping in cone metric spaces.

Keywords: Cone Metric Space, Complete Cone Metric Space, Cauchy Sequence, Compatible Mapping, Fixed Point.

Introduction

The first author and Junck (5) established the existence of coincidence points and common fixed point for mappings define on cone metric space in this section we obtain several fixed point theorem for mappings with applying conditions, defined on a cone metric spaces.

Let E be a real Banach space. A subset P of E is called a cone. If and only of -

1. P is closed non empty and P ≠ {0}
2. a, b $\in$R, a, b $\geq$ o, x, y $\in$ P => ax + by $\in$P,
3. P $\cap$ (-P) = {0}

Give a cone P$\in$E, we define a partial ordering $\leq$ with respect to P by x $\leq$ y off y - x $\in$P. A cone P is called normal and K70 such that for all x, y $\in$E.

=> 11 x 11 < K 11 y 11 ; 0 < x < y .... (1.1)

Preliminaries

Definition (2.1)

Let X be a non empty set, suppose that the d : X x X -> E ratiesties.

1. 0 $\leq$ d (x, y) x, y $\in$ x
2. d (x, y) = d (y, n) x, y $\in$ x and
   d (x, y) = 0 if x = y
3. d (x, y) $\leq$ d (x, z) + d (z, y) V x, y, z $\in$ x

Then d is called a cone metric on X and (x, d) is called a cone metric space.

Definition (2.2)

Let (x, d) be a cone metric space. We say that {xn} is.

1. a Cauchy sequence if for every C in E with C > 0 there is N such that for all n > N d (x_n, x_m) < C.
2. a convergent sequence of for every C in E with C > 0 there is N such that for all n > N cd (x_n, n) < C for some x in X.

A cone metric space X is said to be complete of every Cauchy sequence in X is convergent in X. It is know that {x_n} converges to x < X off d(x_n, x) -> 0 as n -> ∞.

Definition (2.3)

Let f and g be self maps of a set X if W = f (x) = g (x) for some x in X then x is called a coincidence point of coincidence of f and g.

Definition (2.4)

Let f and g be two self maps of a metric space X. Then f and g said to be compatible of limit d (fgx_n, gfx_n) whenever {x_n} is a sequence such that lim fx_n = lim gx_n = t <- x.

Main Result

Theorem

Let (x, d) be a complete cone metric space and P be a normal cone with normal constant K. Suppose that the mappings F and T are two self maps of X with T (x) $\leq$ F (x) satisfying the conditions.

d (Tx, Ty) $\leq$ d (Fx, Fy) + max {d (Fx, Tx), d (Fy, Ty)} +
c {d (Fx, Ty) + d (Fy + Tx)} I

For all x, y $\in$ x where a, b, c > 0 and a + b + 2c < I

Then F and T have a coincidence point in X. Moreover, Coincidence value is unique. i.e. Fp = Fq whenever Fp = Tp and = Tq (p, q $\in$ x)

Proof

Let xo $\in$ x since T (x) < F (x)

We choose x, so that $y_1 = Fx_1 = Tn_0$. in general choose x_{n+1} such that $y_{n+1} = Fx_{n+1} = Txn$ from (I).

$d(y_{n+1}, y_{n+2}) = d(Tx_n, Tx_{n+1})$

$\leq ad(Fx_n, Fx_{n+1}) + b \max \{d(Fx_n, Tx_n), d(Fx_{n+a}, Tx_{n+1})\} + c\{d(Fx_n, Tx_{n+1}) + d(Fx_{n+1}, Tx_n)\}$

$= ad(y_n, y_{n+1}) + b \max \{d(y_n, y_{n+1}), d(y_{n+1}, y_{n+2})\} + c\{d(y_n, y_{n+2}) + d(y_{n+1}, y_{n+2})\}$

$= ad(y_n, y_{n+1}) + b\, d\{d(y_{n+1}, y_{n+2})\} + c\{d(y_n, y_{n+2}) + d(y_{n+1}, y_{n+2})\}$

($\therefore$ for some n $d(y_{n+1}, y_{n+2}) > d(y_n, y_{n+1})$)

=> $[1 - (b + c)]\, d(y_{n+1}, y_{n+2}) = (a + c)\, d(y_n, y_{n+1})$

=> $d(y_{n+1}, y_{n+2}) = (a + c) / (1 - (b + c))\ d(y_n, y_{n+1})$

=> $d(y_{n+1}, y_{n+2}) \leq \delta\, d(y_n, y_{n+1})$

Where $\delta = (a + c) / (1 - (b + c)) < 1$ Similarly it can be show that

$d(y_{n+2}, y_{n+3}) \leq \delta\, d(y_{n+1}, y_{n+2})$

$\leq \delta^{n+1} d(y_0, y_1)$

Now for any m > n

$d(y_m, y_n) \leq d(y_n, y_{n+1}) + d(y_{n+1}, y_{n+2}) + \ldots\ldots\ldots\ldots + d(y_{m-1}, y_m)$

$\leq \{\delta^n + \delta^{n+1} + \ldots\ldots\ldots + \delta^{m-1}\}\, d(y_1, y_0)$

$\leq \delta^n / (1 - \delta)\ d(y_1, y_0)$

=> $\| d(y_m, y_n) \| \leq \delta^n / (1 - \delta)\ K \| d(y_1, y_0) \|$

(From 1.1)

=> $d(y_n, y_m) \to 0$ as $n \to 0$

Hence $\{y_n\}$ is a Cauchy sequence.

Since X is complete then there exists Z in X such that P = F (z) from I

$d(Fx, Tz) < d(Fz, Fx_{n+1}) + d(Fx_{n+1}, Tz)$

$\leq d(Fz, Fx_{n+1}) + d(Tx_n, Tz)$

$\leq d(Fz, Fx_{n+1}) + a\, d(Fx_n, Fz) + b \max \{d(Fx_n, Tx_n),$
$d(Fx, Tz) + c\{d(Fx_n, Tz) + d(Fz, Tx_n)\}$

$\leq d(Fz, Fx_{n+1}) + a\, d(y_n, P) + b \max \{d(y_n, y_{n+1}),$
$d(Tz, Tz)\} + c\{d(y_n, Tz) + d(Fz, y_{n+1})\}$

Taking the limit $n \to \infty$ yields,

$d(Fz, Tz) \leq (b + c)\, d(Fz, Tz)$

From (1.1)

$\| d(Fz, Tz) \| \leq K(b + c) \| d(Fz, Tz) \|$

Now right hand side of the above inequalities approaches zero as $n \to \infty$. Hence the uniqueness of a limit in a cone metric space implies that F(z) = T(z) = P.

Now we show that F and T have a unique point of coincidence. For in this case $P \in T(x) \leq F(x)$, then to establish uniqueness, suppose that q is another coincidence point of F and T.

From (I) we have

=> $d(Tp, Tq) \leq a\, d(Fp, Fq) + b \max \{d(Fp, Tp),$
$d(Fq, Tq) + c\{d(Fp, Tq) + d(Fq, Tp)\}$

=> $d(Tp, Tq) < (a + 2c)\, d(Tp, Tq)$

This gives $\| d(Tp, Tq) \| = 0$

Which implies that Tp = Tq and hence Fp = Fq from (2.4) F and T have a unique common fixed point.

Corollary

Let (X, d) be complete cone metric space and P be a normal cone with normal constant K. Suppose that T a self map of X satisfy (I) with F = I, the identity map on X. Then T has a unique fixed point.

$d(Tx, Ty) \leq a\, d(x, y) + b \max \{d(x, Tx), d(y, Ty)\} +$
$c\{d(x, Ty) + d(y, Tx)\}$

Where $x, y \in X$ and $a, b, c \geq 0$ and $a + b + 2c < 1$ then T has a fixed point in X.

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